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6th Tutorial to
Econometric Methods and Applications WS 2017/18

Exercise 23 [2.5%]

Given the data set

y	1	0	0	1	1	0	0	1	1	1
x	9	2	5	4	6	7	3	5	2	6

, use suitable software to estimate a probit model and test the hypothesis that x is not influential in determining the probability that y equals one.

Exercise 24 [5%]

Assume that we want to estimate a logit/probit model with only one, binary regressor: $y = F(\beta_0 + \beta_1 x)$ with $x \in \{0, 1\}$.

(a) Show that the likelihood function can be written as

$$L(\beta_0, \beta_1 | x, y) = \prod_{i=1}^{n} \left(F(\beta_0)^{y_i(1-x_i)} \big(1-F(\beta_0)\big)^{(1-y_i)(1-x_i)} F(\beta_0+\beta_1)^{y_i x_i} \big(1-F(\beta_0+\beta_1)\big)^{(1-y_i)x_i} \right)$$

(b) Assuming that the quantities $y_{11} := \sum_{i=1}^{n} y_i x_i$, $y_{01} := \sum_{i=1}^{n} (1-y_i) x_i$, $y_{10} := \sum_{i=1}^{n} y_i (1-x_i)$, and $y_{00} := \sum_{i=1}^{n} (1-y_i)(1-x_i)$ are all positive, derive the formulas for the ML estimates of β_0 and β_1, both for a logit and a probit model.

(c) Which problem(s) occur when not all the quantities defined in (b) are positive?

(d) Under the condition given in (b), give the estimates of the logit and probit model for $P(y = 1|x = 0)$ and $P(y = 1|x = 1)$.

Exercise 25 [2.5%]

Consider the following regressions of labor force participation of women ('lfp') on the number of kids five years old or younger ('kids5'), age, educational attainment ('educ', in years), the number of hours worked by the husband ('hushrs'), the husband's wage ('huswage') and the woman's marginal tax rate ('mtr'):

```
Call:
glm(formula = lfp ~ kids5 + age + educ + hushrs + huswage + mtr,
    family = binomial(link = "logit"), data = Mroz87, x = TRUE)
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.0133  -0.9598   0.4559   0.8574   2.8673

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)  1.989e+01  2.392e+00   8.316  < 2e-16 ***
kids5       -1.362e+00  2.015e-01  -6.760 1.38e-11 ***
age         -7.238e-02  1.232e-02  -5.876 4.21e-09 ***
educ         1.974e-01  4.379e-02   4.508 6.55e-06 ***
hushrs      -1.408e-03  1.953e-04  -7.209 5.62e-13 ***
huswage     -3.910e-01  4.561e-02  -8.573  < 2e-16 ***
mtr         -1.833e+01  2.250e+00  -8.149 3.68e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1029.75  on 752  degrees of freedom
Residual deviance:  821.03  on 746  degrees of freedom
AIC: 835.03

Number of Fisher Scoring iterations: 4


Call:
glm(formula = lfp ~ kids5 + age + educ + hushrs + huswage + mtr,
    family = binomial(link = "probit"), data = Mroz87, x = TRUE)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.2002  -0.9894   0.4658   0.8829   2.9274

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) 10.8090243  1.3262105   8.150 3.63e-16 ***
kids5       -0.8086460  0.1164040  -6.947 3.73e-12 ***
age         -0.0425951  0.0071720  -5.939 2.87e-09 ***
educ         0.1176108  0.0254638   4.619 3.86e-06 ***
hushrs      -0.0007690  0.0001097  -7.009 2.40e-12 ***
huswage     -0.2135831  0.0253011  -8.442  < 2e-16 ***
mtr         -9.8705119  1.2448218  -7.929 2.20e-15 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)
```

```
    Null deviance: 1029.75  on 752  degrees of freedom
Residual deviance:  826.55  on 746  degrees of freedom
AIC: 840.55

Number of Fisher Scoring iterations: 5
```

(a) What is the difference between the two estimated models?

(b) Which probability of labor force participation do the models predict for a 32 year old woman with one kid aged five or younger, twelve years of education, and a marginal tax rate of 0.7215, whose husband worked 2708 hours for a wage of 4.0288 dollars?

(c) Calculate, for both models, the marginal effect of additional education for the woman described in (b).

(d) Repeat (b) and (c) for a 37 year old woman with no kids aged five or younger, sixteen years of education, and a marginal tax rate of 0.6915, whose husband worked 2670 hours for a wage of 3.4277 dollars.